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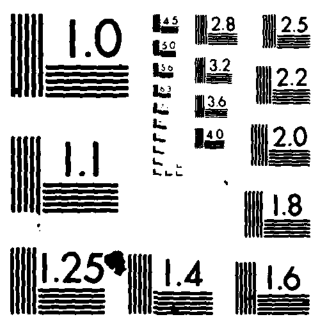

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LEVEL IV

INTERIM SCIENTIFIC REPORT

Air Force Office of Scientific Research Grant AFOSR-79-0127

Period: 30 September 1979 through 29 September 1980

Title of Research: Foundations of Eigenvalue Distribution Theory for General & Nonnegative Matrices, Stability Criteria for Hyperbolic Initial-Boundary Value Problems, Exact Eigenvalue Computations on the ILLIAC IV

Principal Investigators: M. Marcus
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This document summarizes 46 research papers which have either appeared or will appear in major refereed publications. Highlights of some of the results include: (1) Convenient stability criteria for difference approximations to hyperbolic initial boundary value problems have been obtained. The new criteria are given in terms of the boundary conditions and are independent of the basic scheme. (2) The known stability condition for the multi-dimensional Lax-Wendroff scheme has been improved. (3) Norm properties of C-numerical radii were studied. In particular, multiplicativity factors were obtained for

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the case where C is a normal matrix. Also, the following computer codes written for the ILLIAC IV were completed: (1) a program to determine the rank and a maximal nonvanishing subdeterminant of an integral m X n matrix; (2) a program to determine the null space of an integral m X n matrix; and (3) a program to determine all the rational solutions of an arbitrary integral linear system of equations.

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ABSTRACT

Marvin Marcus: PROBLEMS IN DETERMINING REGIONS IN THE COMPLEX PLANE CONTAINING EIGENVALUES OF A LINEAR OPERATOR: VARIATIONS ON THE HAUSDORFF-TOEPLITZ NUMERICAL RANGE.

Research either appeared or in press during the period Oct. 1, 1979 - Sept. 30, 1980 covered the following topics: (1) values assumed by a product of quadratic forms; (2) inequalities relating eigenvalues and non-principal subdeterminants of an arbitrary complex matrix; (3) lower bounds for the spread and Hilbert norm of an arbitrary complex matrix; (4) numerical computations relating eigenvalues and non-principal subdeterminants; (5) a simplification of the Goldberg-Straus proof of norm properties of C-numerical radii; (6) definition and analysis of a new class of generalized operator norms containing the classical work of Fan, Weyl, von Neumann and others; (7) the analysis of certain linear groups defined by tensor identities; (8) an extension of the Cauchy-Schwarz inequality.

The purpose of this interim report is to summarize the status of the work of M. Marcus, sponsored by the Air Force office of Scientific Research, covering the general topic of eigenvalue analysis of matrices and compact operators. The presentation is in reverse chronological order beginning with a description of research recently completed and currently underway.

1. The numerical range. The classical Hausdorff-Toeplitz numerical range [1, 2] of an operator A is the set of complex numbers $W(A) = \{(Ax, x) \mid \|x\| = 1\}$. The important facts about $W(A)$ from the standpoint of eigenvalue localization are: if A is normal then $W(A)$ is the convex polygon spanned by the eigenvalues of A; $W(A)$ is convex for any A; $W(A)$ contains the eigenvalues of A for any A; any non-differentiable point (i.e., cusp point) on the boundary curve of $W(A)$ is an eigenvalue of A. Included with this report are several transparencies of numerical ranges (and higher ranges) that were computed using FORTRAN routines developed at UCSB and run on the AS/6.

In [3] and [4, 5, 6] von Neumann and later Fan introduced what Halmos [7] subsequently called the higher numerical range: $W_k(A) = \{z \mid z = \sum_{j=1}^{k} (Ax_j, x_j), x_1, \ldots, x_k \text{ o.n.}\}$. That is, $W_k(A)$ is the set of sums of values of quadratic forms over a fixed number, k, of varying orthonormal vectors (i. e., mutually perpendicular unit vectors). $W_k(A)$ shares certain properties in common with $W(A)$: $W_k(A)$ contains all sums of k eigenvalues of A; $W_k(A)$ is convex; any non-differentiable point (i.e., cusp point) on the boundary curve of $W_k(A)$ is a sum of k eigenvalues of A. (Some of these facts were discovered at UCSB in work sponsored by AFSC). Many mathematicians have contributed to research stemming from the original papers of von Neumann and Fan: Au-Yueng, Bauer, Bellman, Davis, De Pillis, Deprima, Deutsch, Djokovic, Donoghue, Farnell, Fillmore, Givens, Goldberg, Halmos, Henrici, Horn, Johnson, Kato, Magnus, Mirsky, Olkin, Parker, Pearcy, Putman, Schneider, Givens, Goldberg, Stampfli, Straus, Taussky-Todd, Thompson, Westwick, Wielandt, Williams, Zenger.

Research currently underway by M. Marcus is related to the recent work of Goldberg and Straus on numerical radii. As Goldberg points out in his report, this concept is a valuable tool in stability analysis of finite difference approximations for hyperbolic systems of P.D.E. The numerical radius of A denoted by $r(A)$, is simply the distance of the farthest point in $W(A)$ from the origin. In a seminar in the early summer of 1980 involving three of the P.I.'s on this grant, Marcus introduced the following definition: let $G = [G_1 : G_2 : G_3]$ be a $k \times 3k$ matrix in which G_1 and G_2 are positive definite. Two sets of vectors $x_1, \ldots, x_k$, and $y_1, \ldots, y_k$ are G-vectors if $[(x_i,x_j)] = G_1$, $[(y_i,y_j)] = G_2$, $[(x_i,y_j)] = G_3$. The G-<u>numerical range</u>, denoted by $W_G(A)$, is defined as the totality of numbers $\sum_{i=1}^{k} (Ax_i, y_i)$ in which the x_i and y_i vary over all sets of G-vectors. Of course, if $G_1 = G_2 = G_3 = I_k$ then $W_G(A)$ specializes to the higher numerical range; if $G_1 = G_2 = 1$, $G_3 = 0$ then $W(A)$ has been used by Mirsky to study the spread of A (i.e., the maximum distance between two eigenvalues of A). A great many obviously important questions immediately arise about $W_G(A)$: when is $W_G(A)$ non-empty, i. e., when do there exist sets of G-vectors ? when is $W_G(A)$ convex? ; how does the choice of G effect various functions of the eigenvalues of A that lie in $W_G(A)$?; can an effective computer routine be developed for plotting $W_G(A)$?; can simple functions of the eigenvalues of A be found in the intersection of several $W_G(A)$ for various choices of G ?; if the G - numerical radius, $r_G(A)$, is defined as the distance of the farthest point in $W_G(A)$ to the origin then is $r_G(A)$ a matrix norm ?. In the first two of a series of papers currently being prepared M. Marcus and M. Sandy (a Phd student) we have completely answered the first two and the last of these questions. Some of these results will be presented at the Annual Meeting of the American Mathematical Society, Jan. 7 - 11, 1981.

The proof that $r_G(A)$ is a norm is related to the Goldberg-Straus work on C - numerical radii, and, in fact, substantial simplifications in the proof of their principal result have just recently been obtained [8].

In [9] Marcus solved a research problem posed by A. Abian in the Notices of the American Mathematical Society: what are the conditions on P_1, P_2, Q_1, Q_2 that

$$(P_1u, u)(P_2v, v) \geq (Q_1u, v)(Q_2v, v)$$

for all vectors u and v. This inequality has just recently been reconsidered under the present grant as a special case of operators on the $m^{\underline{th}}$ tensor space of the form

$$L = P_1 \otimes \ldots \otimes P_m - Q_1 \otimes \ldots \otimes Q_m \sigma$$

in which σ is a permutation operator corresponding to a permutation in S_m with no fixed points (the case $m = 2$ is the Abian problem)
The problem is to determine conditions on $P_1, \ldots, P_m$, $Q_1, \ldots, Q_m$ for L to be decomposably non-negative, i.e., for $(Lu_1 \otimes \ldots \otimes u_m, v_1 \otimes \ldots \otimes v_m) \geqq 0$ to be satisfied for all vectors $u_1, \ldots, u_m$, $v_1, \ldots, v_m$. This problem is now completly solved and currently being prepared for publication [10]. Along these lines M. Marcus and Bo-Ying Wang introduced the set $W^{\perp}(L)$, where L is a linear operator on the

tensor space $\overset{k}{\underset{1}{\otimes}} V$, and the indicated set consists of all complex numbers of the form $(L x_1 \otimes \dots \otimes x_k, x_1 \otimes \dots \otimes x_k)$, in which $x_1, \dots, x_k$ are o.n. vectors. This formulation includes all known versions and extensions of the classical Hausdorff-Toeplitz numerical range. Some of the results obtained recently are: $W^{\perp}(L)$ in the origin iff $L = 0$; if $\dim V > 2$ then $W^{\perp}(A \otimes B) \subset R$ iff $A \otimes B$ is hermitian; if $L = A_1 \otimes \dots \otimes A_k$, then $W^{\perp}(L)$ is a single non-zero point iff L is a non-zero multiple of the identity. All of these results are parts of a larger plan to study the structure of the set

$$W^G(L) = \{(L x, y) \mid x = x_1 \otimes \dots \otimes x_k, \quad y = y_1 \otimes \dots \otimes y_k \text{ and the } x_i \text{ and } y_i \text{ are G-vectors as defined above}\}$$

The set $W^G(L)$, the G -<u>decompable numerical range</u>, appears to us to be the proper setting in which to incorporate most of the results on variations of the numerical range, relations between eigenvalues and quadratic forms etc. that have appeared in the 167 papers published on this subject beginning with the pioneering work of Hausdorff, Toeplitz, von Neumann and Fan.

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Marvin Marcus

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8. M. Marcus, M. Sandy, A generalization of the higher numerical range, Linear and Multilinaer Algebra (in preparation)

9. M. Marcus, Bo-Ying Wang, Some variations on the numerical range II (in preparation)

ATTACHMENT TO MARCUS REPORT

In transparency I the ordinary numerical range

$$W_1(A) = \{(Ax,x) \mid \|x\| = 1\}$$

of the indicated 4 X 4 matrix is exhibited. In transparency II the plot shows that the ordinary numerical range $W_1(A)$ coincides with the convex polygon $P_1(A)$ spanned by the eigenvalues. However, the 2nd numerical range,

$$W_2(A) = \{(Ax_1,x_1) + (Ax_2,x_2) \mid x_1,x_2 \text{ orthonormal}\}$$

strictly contains the convex polygon $P_2(A)$ spanned by all sums $\lambda_i + \lambda_j$, $i < j$, of the eigenvalues of A, the indicated 5 X 5 matrix. In transparency III both $W_1(A) = P_1(A)$ and $W_2(A) = P_2(A)$ for the indicated 7 X 7 matrix A. However the 3rd numerical range,

$$W_3(A) = \{(Ax_1,x_1) + (Ax_2,x_2) + (Ax_3,x_3) \mid x_1,x_2,x_3\}$$

strictly contains the convex polygon $P_3(A)$ spanned by all sums $\lambda_i + \lambda_j + \lambda_k$, $i < j < k$, of the eigenvalues of A.

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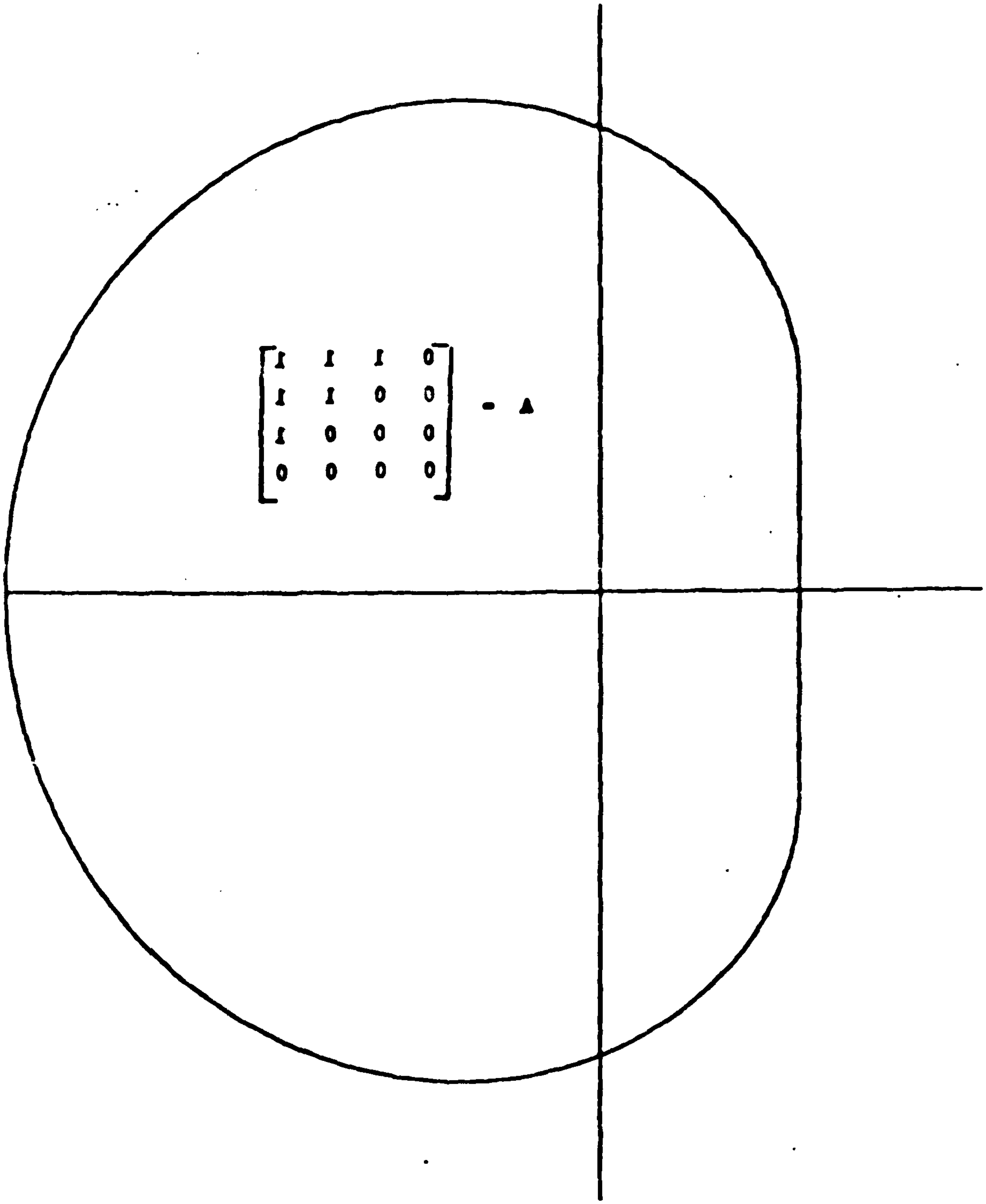
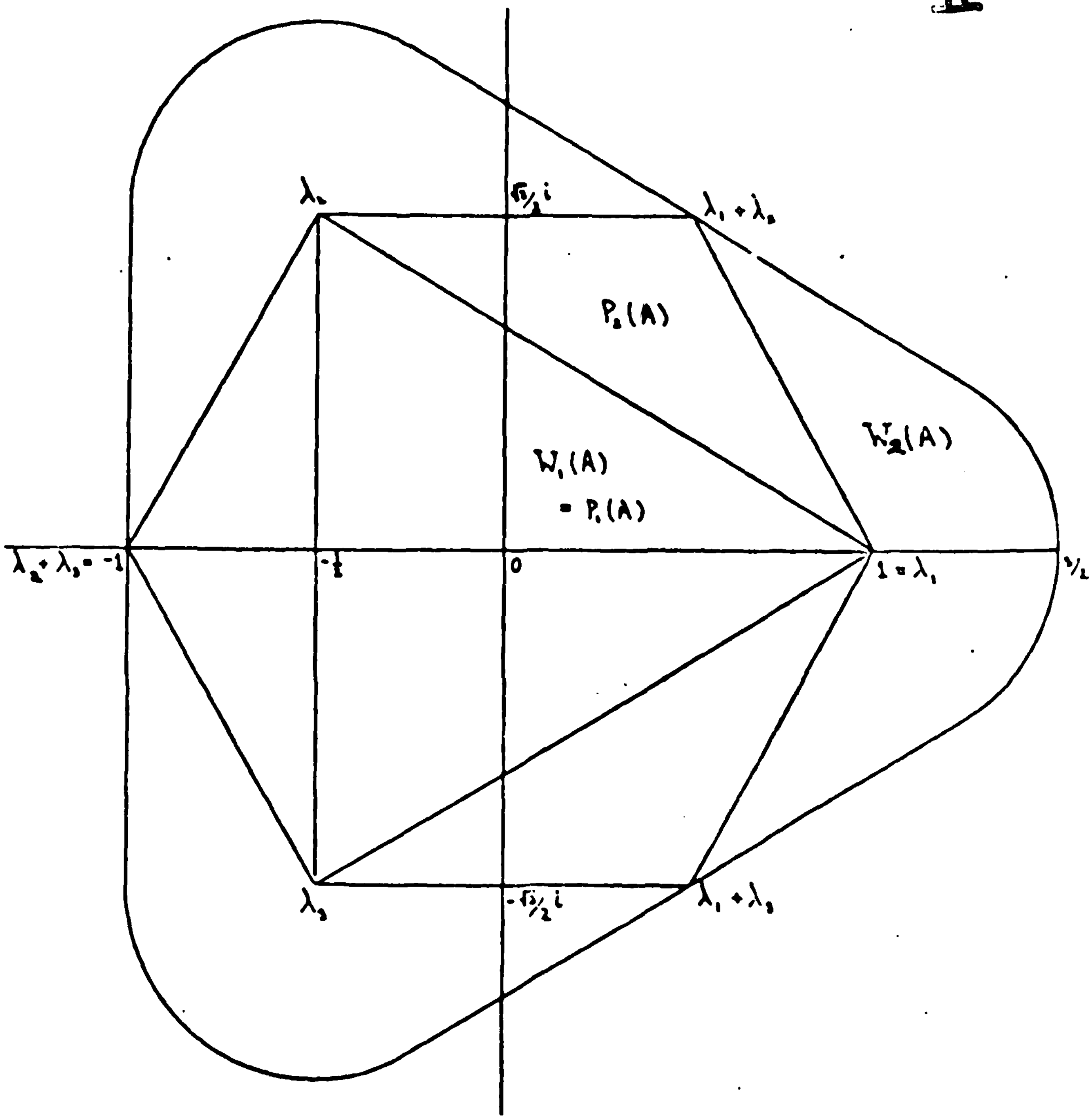


$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = A$$

II

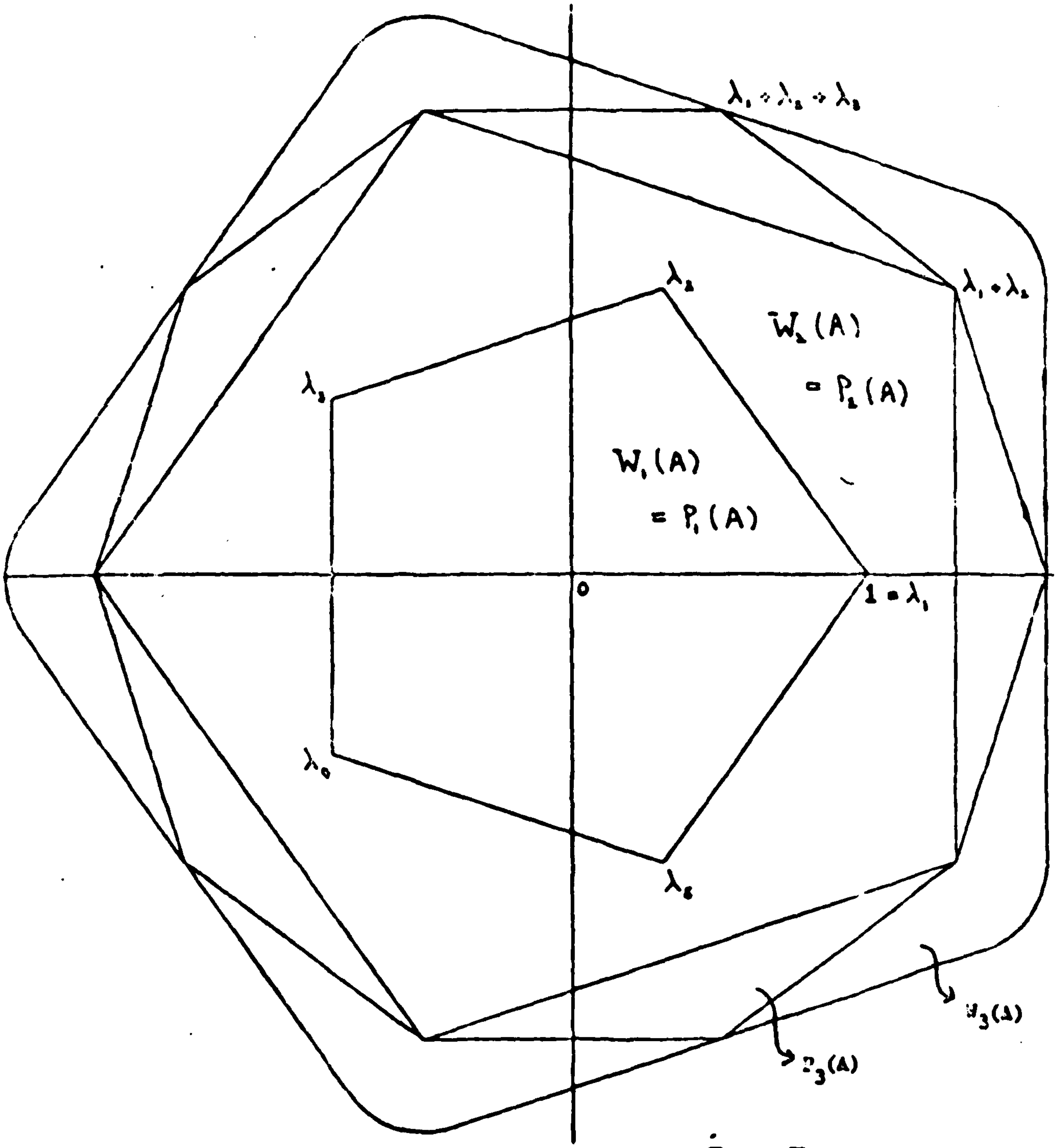


$$A = \mathrm{diag}(\lambda_1, \lambda_2, \lambda_3) \dotplus \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix}$$

$$\lambda_k = e^{2(k-1)\pi\, i/3}, \quad k = 1,\ 2,\ 3$$

$$c = 1 = 2 \cos \frac{\pi}{3}$$

III



$$A = \operatorname{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5) \dotplus \begin{bmatrix} 0 & c \\ 0 & 0 \end{bmatrix}$$

$$\lambda_k = e^{2(k-1)\pi i/5}, \quad k = 1, 2, 3, 4, 5$$

$$c = .618 \doteq 2\cos\frac{2\pi}{5}$$

ABSTRACT

M. Goldberg: PROBLEMS IN STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC SYSTEMS

Research completed includes the following: (1) Convenient stability criteria for difference approximations to hyperbolic initial-boundary value problems have been obtained. The new criteria are given in terms of the boundary conditions and are independent of the basic scheme. (2) The known stability condition for the multi-dimensional Lax-Wendroff scheme has been improved. (3) Norm properties of C-numerical radii were studied. In particular, multiplicativity factors were obtained for the case where C is a normal matrix.

M. Goldberg: PROBLEM IN STABILITY ANALYSIS OF FINITE DIFFERENCE SCHEMES FOR HYPERBOLIC SYSTEMS.

The purpose of this interim report is to summarize my Air Force sponsored research in stability analysis of finite difference approximations of hyperbolic partial differential systems and related topics, during the period October 1979 – September 1980.

1. Convenient Stability Criteria for Hyperbolic Initial-Boundary Value Problems.

In the past year E. Tadmor and I, [6], have succeeded in extending the results of [5] to obtain easily checkable stability criteria for difference approximations of initial-boundary value problems associated with the linear hyperbolic differential systems

$$\partial \underset{\sim}{u}(x,t)/\partial t = A\partial \underset{\sim}{u}(x,t)/\partial x + B\underset{\sim}{u}(x,t) + \underset{\sim}{f}(x,t), \quad x \geq 0, \quad t \geq 0 \quad ,$$

where $\underset{\sim}{u}(x,t)$ is the unknown vector, A a Hermitian matrix, B an arbitrary matrix and $\underset{\sim}{f}(x,t)$ a given vector. The difference approximations consist of arbitrary basic schemes -- explicit or implicit, dissipative or unitary, two-level or multi-level -- and boundary conditions of a rather general type.

In the first step of our stability analysis we prove that the approximation is stable if and only if the scalar outflow components of its principal part are stable. This reduces the global stability question to that of a scalar, homogeneous, outflow problem.

Investigating the stability of the reduced problem, our main results are restricted to the case where the boundary conditions are translatory, i.e., determined at all boundary points by the same coefficients. Such boundary conditions are commonly used in practice; and in particular, when the numerical boundary consits of a single point the boundary conditions are translatory by definition.

The main stability criteria for the translatory case are given essentially in terms of the boundary conditions. Such scheme-independent criteria eliminate the need to analyze the intricate and often complicated interaction between the basic scheme and the boundary conditions; hence providing convenient alternatives to the well known stability criterion of Gustafsson, Kreiss and Sundström [8].

We assume that the basic scheme is stable for the pure Cauchy problem and that the approximation is solvable. Under these basic assumptions -- which are obviously necessary for stability -- we obtain, for example, that the reduced problem is stable if the (translatory) boundary conditions are solvable and staisfy the von Neumann condition as well as an additional simple inequality. If the basic scheme is unitary it is also required that the boundary conditions be dissipative.

Having the new stability criteria, we studied several examples. First, we reestablish the known fact that if the basic scheme is two-level and dissipative, then outflow boundary conditions determined by horizontal extrapolation always maintain stability. Surprisingly, we show that this result is false if the basic scheme is of more than two levels. Next, for arbitrary multi-level dissipative basic schemes we find that if the outflow boundary conditions are generated, for example, by oblique extrapolation, by the Box-Scheme, or by the right-sided Euler scheme, then overall stability is assured. Finally, for general basic schemes (dissipative or unitary) we show that overall stability holds if the outflow boundary conditions are determined by the right-sided explicit or implicit Euler schemes. These examples incorporate many special cases discussed in recent literature [1, 5, 8, 9, 10, 11, 13, 14].

It should be pointed out that there is no difficulty in extending our stability criteria to cases with two boundaries. In fact, if the corresponding left and right quarter-plane problems are stable, it is known [8] that the original two-boundary problem is stable as well.

At present, Tadmor and I are studying further stability criteria which result from the one obtained recently. We also intend to investigate malposed boundary conditions for hyperbolic inflow problems. We expect to complete this work by the summer of 1981.

2. Numerical Radii and Matrix Norms.

(i) The importance of the numerical radius as a tool in stability analysis of finite difference approximations for multi-dimensional hyperbolic systems with constant coefficients is well known. In a forthcoming work with E. Tadmor, [7], we use this tool to improve the known stability condition for the Lax-Wendroff scheme in dimension $d \geq 2$. We show that if the differential system is

$$\partial \underset{\sim}{u}(x_1,\ldots,x_d,t)/\partial t = \sum_{j=1}^{d} A_j\, \partial \underset{\sim}{u}(x_1,\ldots,x_d,t)/\partial x \quad ,$$

A_j being fixed coefficient matrices, then the corresponding Lax-Wendroff scheme is stable if and only if

$$\Delta t \leq \frac{1}{d}\left[\sum_{j=1}^{d} \frac{\rho^2(A_j)}{(\Delta x_j)^2}\right]^{-1/2}$$

where $\rho(A_j)$ denotes the spectral radius of A_j. This is a considerable improvement of the original Lax-Wendroff criteria [12],

$$\Delta t \leq \frac{1}{d^{3/2}} \cdot \left[\max_j \frac{\rho(A_j)}{\Delta x_j}\right]^{-1}$$

(ii) In two recent papers [2,3], E.G. Straus and I have studied norm properties of C-numerical radii which constitute the following generalization of the classical radius: Given $n \times n$ matrices A, C, the C-numerical radius of A is the non-negative quantity

$$r_C(A) = \max\{|\operatorname{tr}(CU^*AU)| : U \text{ unitary}\} .$$

Evidently r_C is a semi-norm on the algebra of $n \times n$ matrices, and for $C = \operatorname{diag}(1,0,\ldots,0)$ it reduces to the classical radius $r(A)$.

We prove that r_C is a generalized matrix norm if and only if C is not scalar and $\operatorname{tr} C \neq 0$. This provides a large family of new norms which might prove useful in application. Also, the result agrees with the well known fact that the classical radius is a generalized matrix norm.

A significant disadvantage of the classical radius is that it is not multiplicative, i.e., it is not an ordinary matrix norm. This led us to consider arbitrary generliazed matrix norms N and characterize all positive constants ν for which νN is multiplicative.

we call such ν "multiplicativity factors" for N. Applying our results to $r_C(A)$ we found multiplicativity factors in the case where C is Hermitian. In particular, we showed that νr is multiplicative if and only if $\nu \geq 4$, independently of the dimension of the space.

Currently, Straus and I [4] are examining multiplicativity factors for arbitrary C-radii. Related topics are under continuing study as well.

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5. M. Goldberg and E. Tadmor, Scheme-independent stability criteria for difference approximations of hyperbolic initial-boundary value problems. II, Mathematics of Computation, to appear.

6. M. Goldberg and E. Tadmor, Numerical radius and multi-dimensional Lax-Wendroff schemes, in preparation.

ABSTRACT

M. Newman: THE APPLICATIONS OF NUMBER THEORY TO COMPUTATION

Research completed in the last period includes the following: (1) The production of an ILLIAC IV program to determine the rank and a maximal nonvanishing subdeterminant of an integral m X n matrix. (2) The production of an ILLIAC IV program to determine the null space of an integral m X n matrix. (3) The production of an ILLIAC IV program to determine all the rational solutions of an arbitrary integral linear system of equations.

M. NEWMAN: The applications of number theory to computation.

The purpose of this report is to summarize my AIR FORCE sponsored research on the application of number theory (in particular modular arithmetic) to problems of computation, using the parallel processing features of ILLIAC IV.

The principal objective of my research is to apply number-theoretic ideas to problems of numerical analysis and computation. In particular, applications of modular arithmetic are being made to the following topics:

(1) The exact solution of an integral system of linear equations, and the exact computation of the determinant of the system;

(2) The determination of the exact inverse of an integral matrix using minimal storage;

(3) The determination of the rank and a basis for the null space of an inegral matrix;

(4) The determination of all rational solutions of an integral system of linear equations;

(5) The determination of the eigenvalues of a rational symmetric triple diagonal matrix to any desired accuracy;

(6) The computation of the permanent of a matrix.

Programs to perform (1), (2), (3), and (4) which take full advantage of parallel computation have been prepared for ILLIAC IV. ILLIAC IV programs for (5) and (6) are in process of preparation.

The primary objective of the research undertaken in the last period was to produce ILLIAC IV programs to determine the rank and a maximal nonvanishing subdeterminant of an integral $m \times n$ matrix. This program was then used as part of another program which determines a basis for the null space of an integral $m \times n$ matrix. Finally, these programs were used to prepare a master program which finds all the rational solutions of an arbitrary integral linear system of equations. All of these programs are available as ILLIAC IV programs, and are quite efficient. For example, any system of m equations is n unknowns with $m \leq 40$, $n \leq 40$, can be processed in at most 30 seconds. The comparable time for a serial machine would be inordinately large; perhaps 3 hours.

I have had invaluable help from the staff of the Institute for Advanced Computation at Sunnyvale, and also from Paol Nikolai, at Wright-Patterson Air Force Base in Dayton, Ohio.

My research assistent John Shure has written a Master's Thesis on this topic (copy enclosed). A detailed account of this project will appear in Math Comp. at some future time.

M. Newman: PUBLICATIONS

October 1, 1979, to date

1. Eqivalence without determinantal divisors, Linear and Multilinear Algebra 7, 107-109 (article), (1979).

2. The use of integral operators in number theory (with C. Ryavec and B. N. Shure), J. Functional Analysis 32, 123-130 (article), (1979).

3. A note on conspectral graphs (with C. R. Johnson), J. Comb. Theory, B28, 96-103, (1980).

4. On a problem suggested by Olga Taussky-Todd, Illinois J. Math. 24, 156- 158, (1980).

5. Positive definite matrices and Catalan Numbers (with F. T. Leighton), Proc. Amer. Math. Soc. 79, 177-181, (1980).

6. Matrices of finite period and some special linear equations, Linear and Multilinear Algebra, 8, 189-195, (1980).

7. Gersgorin revisited. Linear Algebra and its Applications 30, 247-249, (1980).

8. A surprising determinantal inequality for real matrices (with C. R. Johnson), Math. Ann. 247, 179-180, (1980).

9. A radical Diophantine equation, to appear in J. Number Theory.

10. Determinants of Abelian group matrices (with M. Mahoney), to appear in Linear and Multilinear Algebra.

11. Determinants of circulants of prime power order, to appear in Linear and Multilinear Algebra.

ABSTRACT

R.C. Thompson: SINGULAR VALUES

Singular values and invariant factors have continued to be investigated. Applications to the matrix valued triangle inequality are studied as well as to products of matrix exponentials.

The singular values of a matrix A are the eigenvalues of $(AA^*)^{\frac{1}{2}}$ where A* is the complex conjugate transpose of A. These elusive non-negative numbers are among the most important and useful quantities that may be computed for a matrix. For example, they are intimately connected with the popular "generalized inverse", they have many applications in numerical linear algebra, and their properties are best comprehended in Lie-group and Lie-algebra theoretic terms. A discovery of Thompson a few years ago is that number theory, particularly the study of the invariant factors of matrices with integer entries, affords an excellent guide to the properties of singular values. Moreover, since the number theoretic investigations are usually easier than the corresponding singular value analysis, it is sound practice to precede each investigation of singular values with a number theoretical study. A large part of the effort in the past year has been expended in this manner, preparing the way for a major understanding in the study of singular values by doing certain of the (<u>easier</u> but still <u>very</u> difficult) number theoretical studies of invariant factors.

Thompson has conjectured that, if A and B are matrices with real or complex entries, then

$$e^A e^B = e^{SAS^{-1} + TBT^{-1}} \tag{1}$$

for appropriate matrices S and T (dependent on A and B). This conjecture is proposed only for A and B sufficiently near zero, since Thompson has demonstrated its falseness for matrices A,B with large entries. It has also been shown by Thompson, using the Campbell-Baker-Hausdorff formula, that this conjecture is formally correct, in that infinite series exist giving S and T which work in (1). The only difficulty is that the series cannot be proved to converge, and Thompson believes that they do not converge. Thus an alternative strategy is needed to prove (1), and the first phase of this strategy is under way. The strategy is to analyze how the similarity invariant factors behave when complex matrices add or multiply, then use the results of this analysis to prove (1). The first, nearly completed, phase is the same question for the invariant factors of matrices with integer (rather than complex) entries. Striking results have been obtained, a general conjecture formulated, and proved in many cases. The results of this initial phase will be incorporated in paper 15.

The application to singular values will come when the same conjecture (1) is studied for Hermitian matrices A and B. (Thompson mentioned this conjecture at the 1980 Auburn (Ala.) matrix conference. The superb mathematician, Friedland, was present at this conference and voiced his opinion that the conjecture was extremely difficult. Others gave the same opinion).

All this work is incorporated in papers 3, 4, 6, 13, 14, 15. It is worth noting that Thompson's efforts on this conjecture have, as a by-product, uncovered some new results pertaining to the Campbell-Baker-Hausdorff formula that is so important in Lie theory.

In another direction, classical stability theory relies, in part, on the study of greatest common right divisors and least common left multiples of polynomial matrices. A number of new results pertaining to these questions were found in paper 12.

A result of Thompson a few years ago is the matrix valued triangle inequality: if $|A| = (AA^*)^{\frac{1}{2}}$ is a matrix valued absolute value for matrix A, then Thompson's result is the

$$|A+B| \leq U|A|U^* + V|B|V^*$$

for suitable unitary matrices dependent on A and B. Following Thompson's suggestion, his functional analysis colleague, C.A. Akemann, has proved an infinite dimensional version valid for Hilbert space operators. Work is under way on the corresponding p-adic (paper 5) and quaternon results.

Paper 10 is really a contribution to harmonic analysis: it generalizes, and points the way to many further generalizations, of certain matrix theory embedding theorems that have been found useful in harmonic analysis.

Paper 12 reproves by "interlacing inequalities" certain results proved by others using Lie methods. As such, it represents a continuing attempt by Thompson to fit his investigations into a Lie-theoretic framework.

Research is vigorously continuing.

R. C. Thompson

PUBLICATIONS

October 1, 1979 - September 30, 1980

The following delineates the status of papers published or in preparation.

(a) Published papers. All of these represent work completed prior to October 1979, but published since October 1979.

1. Invariant factors under rank one perturbations, Canadian Journal of Math., V. 32 (1980), pp. 240-245.

2. The congruence numerical range, Linear and Multilinear Algebra, V. 8 (1980), pp. 197-206.

3. Invariant factors of complementary minors of integral matrices, Houston Journal of Math., V. 5 (1979), pp. 421-425.

4. The Smith invariants of a matrix sum, Proc. Am. Math. Soc., V. 78 (1980), pp. 162-164.

(b) Papers in preparation. Some of these are manuscripts substantially completed prior to October 1979, but not submitted to journals. All will eventually be published, although several will likely incorporate updating in view of the probable outcome of current (Oct. '80) research.

5. p-adic matrix valued inequalities, in preparation, 12 typed pages.

6. A matrix exponential formula, in preparation, 10 typed pages.

7. The Smith form, the inversion rule for 2 X 2 matrices, and the uniqueness of the invariant factors for finitely generated modules, in preparation, 5 typed pages.

8. The Jacobi-Gundelfinger-Frobenius-Iohvidov rule and the Hasse symbol, in preparation, 6 typed pages.

9. A matrix block diagonalization in the presence of a semi-definiteness hypothesis, in preparation, 10 typed pages.

10. Doubly stochastic, unitary, unimodular, and complex orthogonal power embeddings, in preparation, 20 typed pages.

11. Left multiples and right divisors of integral matrices, in preparation, 12 typed pages.

12. Products of simple involutions, in preparation, 15 typed pages.

13. An exponential formula, in preparation, 15 typed pages.

14. A supplement to the Campbell-Baker-Hausdorff-Dynkin formula, in preparation, 3 typed pages.

15. The invariant factors of a matrix sum II, still being researched.

ABSTRACT

Henryk Minc: INVERSE ELEMENTARY DIVISOR PROBLEMS FOR NONNEGATIVE MATRICES.

BOUNDS FOR PERMANENTS.

Research during the period included:

(1) Study of the problem: does there exist a doubly stochastic matrix with the same spectrum as a given stochastic matrix but with arbitrarily prescribed elementary divisors consistent with the spectrum. The question was answered in the negative in general, and in the affirmative in case the given matrix is positive and diagonalizable, and also in case the prescribed spectrum consists of only two distinct eigenvalues.

(2) The same question as in (1) for general nonnegative matrices was answered in the affirmative in case the given matrix is positive and diagonalizable.

(3) Bounds for permanents were obtained. In order to improve the known bounds for the 3-dimensional dimer problem, a large number of permanents of (0, 1) - circulants has been computed.

Henryk Minc: (I) INVERSE ELEMENTARY DIVISOR PROBLEMS FOR NONNEGATIVE MATRICES

(1) One of the most important unsolved problems in linear algebra is the <u>inverse eigenvalue problem</u> for nonnegative matrices: to find necessary and sufficient conditions that a given n-tuple of complex numbers be the spectrum of a nonnegative matrix. A parallel problem for doubly stochastic matrices is unsolved as well. The <u>inverse elementary divisor problem</u> for doubly stochastic matrices, the determination of necessary and sufficient conditions that given polynomials be the elementary divisors of a doubly stochastic matrix, contains the inverse eigenvalue problem, and obviously it is also unsolved.

In [4] I showed that doubly stochastic $n \times n$ matrices exist with elementary divisors $\lambda - 1$ and $(\lambda - \alpha)^{e_i}$, $i = 2,\ldots,m$, for any real α, $-\frac{1}{n-1} < \alpha < 1$, and any positive integers $e_2,\ldots,e_m$ whose sum is $n-1$. This result implies that for any $n \geq 3$ there exist doubly stochastic $n \times n$ matrices which have no roots. In [4] I also considered the inverse elementary divisor problem for doubly stochastic matrices modulo the inverse eigenvalue problem: given a doubly stochastic matrix, does there exist a doubly stochastic matrix with the same spectrum and arbitrarily prescribed elementary divisors consistent with the spectrum that do not include $(\lambda - 1)^k$ with $k > 1$ (otherwise the answer would clearly be in the negative). The question is answered in [4] in the negative in general, and in the affirmative in case the given matrix is positive, diagonalizable, and with real eigenvalues.

(2) In [5] I showed that given any positive diagonalizable matrix, there exists a positive matrix with the same spectrum and with any

prescribed elementary divisors consistent with the spectrum. A parallel result for doubly stochastic matrices was also proved, thus extending the result in [4] to diagonalizable positive doubly stochastic matrices with complex, not necessarily real, eigenvalues.

II BOUNDS FOR PERMANENTS

(3) In [2] I obtained bounds for permanents of real matrices. In [1] I utilized Friedland's lower bound for the permanents of doubly stochastic matrices to obtain an improved lower bound for the d-dimensional dimer problem for $d \geqq 4$. In fact I showed that

$$\lambda_d \sim \tfrac{1}{2} \log (2d) - \tfrac{1}{2} .$$

For the all important 3-dimensional case it is known that

$$0.418347 \leqq \lambda_3 < 0.548271,$$

where the lower bound is due to Hemmersley and the upper bound is mine. In order to improve these bounds it is necessary to obtain sharper bounds than the currently known bounds for the permanents of (0,1)-circulants with 6 ones in each row. For this purpose, permanents of some 850 (0,1)-circulants were computed, of orders up to 18 X 18, with 3, 4 or 6 ones in each row. No definite results have been obtained so far.

Henryk Minc

Publications - October 1, 1979 to date

1. H. Minc, An asymptotic solution of the multidimensional dimer problem, Linear and Multilinear Algebra 8 (1980), 235-239.

2. H. Minc, Bounds for permanents and determinants, Linear and Multilinear Algebra 9 (1980), 5-16.

3. H. Minc, Rearrangement inequalities, Proc. Roy. Soc. Edinburgh (to appear).

4. H. Minc, Inverse elementary divisor problem for doubly stochastic matrices (submitted).

5. H. Minc, Inverse elementary divisor problem for nonnegative matrices (submitted).

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